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Challenging one of the Basic Laws of Economics

Should we just accept that the agreed Laws of Economics always apply, regardless of System Complexity?

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Abstract

Part of the job of any CEO/Sr Manager is to give a estimate of how much profit the company will get in the next period. The accountants are responsible for calculating the profits for the previous period. The CEO's job is much more difficult simply because of the variation and uncertainty that relates to the future.

Dr. Eli Goldratt used a simple case study (simple because all the variation and uncertainty was removed to make it deterministic), commonly referred to as the "P&Q example", in his book "The Haystack Syndrome" to show that the rule that will deliver the maximum profit to a company with an internal constraint (demand exceed capacity of one resource) is a very "simple" rule where we prioritize those products with the highest Throughput (Sales Price – Totally Variable Cost) per Constraint Minute

However, with a simple modification to this "P&Q example" that causes more than one resource to be overloaded (demand exceeding capacity of more than one resource) this paper shows that applying the highest T/Cu to prioritize which Product Mix to make, can result in a less than "optimal" solution, to the extent that it can cause the company to make a loss even when a Product Mix exist that can result in a Profit.

This paper shows that for this more complex case which considering the tendency to "balance plants" maybe even the more generic case of more than one resource being overloaded when demand exceed supply capacity, it seems that a Linear Programming algorithm is the only "safe" way to determine the most optimum product mix. But Linear Programming, as the reader might be aware of, is not only a "Complex" solution – it has significant limitations in terms of incorporating the impact of set-ups and batching.

Although the revised P&Q example have provided some new insights (at least to the author) of which variables really govern profitability of Product Mix decisions, it seems as if there is no "simple rule" that will always result in the most "profitable product mix" when more than one resource is overloaded.

However, considering that in reality, there can be significant variation and uncertainty in both the values of the demand and supply constraint's, a sensitivity analysis would have to be done in order to really understand how the most "Profitable" Product Mix would change based on changes in the demand and or supply constraints and also whether there is still a simple rule that can be used for finding a "Good Enough" or "Safe" starting point…

Introduction

The Product Mix Problem we find in the P&Q example is a problem where we need to identify the most "profitable Product Mix" considering that one resource is overloaded – i.e. there is not sufficient capacity to satisfy the full market demand for both Products P & Q. This example shows that if we use the highest Selling Price, the highest total processing time or the highest Margin (or Throughput), we will select a mix that is not optimal, even to the point of causing the factory to run at a loss. The "Rule" that delivers the maximum profit is simply to make that product with the highest Throughput per Constraint Minute (T/Cmin) and use whatever capacity is left to make the product with the 2nd highest T/Cmin and continue until the capacity constraint's capacity is fully consumed.

However, what will happen if in the same example, the processing time was such that more than one resource was overloaded? A Case could be made that this is a more generic problem - since many plants today have been closely "balanced" in capacity in efforts to minimize cost/part - which means that if one resource is overloaded due to an increase in demand, it is likely that at least one more resource might also be overloaded.

So would we still get the maximum profit if we simply applied the same rule as before: "Prioritize those products with the highest T/Cmin where the "Constraint" is the most overloaded resource"?

The Standard P&Q Product Mix Problem (One overloaded Resource)

Original P&Q (Ignoring Capacity Constraint)

	Product P		Product Q		Total	% Utiliz
	Unit	Total	Unit	Total		
Demand	1	**100**	1	**50**		
Sales Price	**$90.00**	$9,000.00	**$100.00**	$5,000.00	$14,000.00	
Variable Cost	**$45.00**	$4,500.00	**$40.00**	$2,000.00	$6,500.00	
Throughput	**$45.00**	$4,500.00	**$60.00**	$3,000.00	$7,500.00	
Operating Expense					$6,000.00	
Net Profit					$1,500.00	
Available Min/Week					2400	
Resource A	15	1500	10	500	2000	83%
Resource B	15	1500	30	1500	**3000**	**125%**
Resource C	15	1500	5	250	1750	73%
Resource D	15	1500	5	250	1750	73%
Totals	60	6000	50	2500	8500	

Original P&Q (Considering Capacity Constraint)

Rule: Prioritize Q over P (based on Highest Throughput or Lowest Total Processing Time)

	Product P		Product Q		Total	% Utilization
	Unit	Total	Unit	Total		
Demand	1	**60**	1	**50**		
Sales Price	**$90.00**	$5,400.00	**$100.00**	$5,000.00	$10,400.00	
Variable Cost	**$45.00**	$2,700.00	**$40.00**	$2,000.00	$4,700.00	
Throughput	**$45.00**	$2,700.00	**$60.00**	$3,000.00	$5,700.00	
Operating Expense					$6,000.00	
Net Profit					**-$300.00**	
Available Min/Week					2400	
Resource A	15	900	10	500	1400	58%
Resource B	15	900	30	1500	**2400**	**100%**
Resource C	15	900	5	250	1150	48%
Resource D	15	900	5	250	1150	48%
Totals	60	3600	50	2500	6100	

As we can see from the above table, following the traditional Cost Accounting Rule of prioritizing based on highest margin (Product Q) or highest Profit/unit (also Product Q) will result in a $300 loss. TOC recommends that we rather prioritize using the product with the highest Throughput per constraint minute (T/Cmin). In the Original P&Q case it is clear that the constraint is Resource B. The table below shows that Product P has the highest T/Cmin.

Original P&Q (Considering Capacity Constraint)

Rule: Prioritize P over Q (based on Highest Throughput per Constraint Min)

	Product P		Product Q		Total	% Utilization
	Unit	Total	Unit	Total		
Demand	1	**100**	1	**30**		
Sales Price	**$90.00**	$9,000.00	**$100.00**	$3,000.00	$12,000.00	
Variable Cost	**$45.00**	$4,500.00	**$40.00**	$1,200.00	$5,700.00	
Throughput	**$45.00**	$4,500.00	**$60.00**	$1,800.00	$6,300.00	
Operating Expense					$6,000.00	
Net Profit					**$300.00**	
Available Min/Week					2400	
Resource A	15	1500	10	300	1800	75%
Resource B	15	1500	30	900	**2400**	**100%**
Resource C	15	1500	5	150	1650	69%
Resource D	15	1500	5	150	1650	69%
Totals	60	6000	50	1500	7500	
T/Cmin (Constraint=B)	**$3.00**		**$2.00**			

The Modified P&Q Product Mix Problem – Scenario with two overloaded Resources

Modified P&Q (Ignoring Capacity Constraints)

Modification: Increase Processing Time for P on Resource D to 25min

	Product P		Product Q		Total	% Utilization
	Unit	Total	Unit	Total		
Demand	1	**100**	1	**50**		
Sales Price	**$90.00**	$9,000.00	**$100.00**	$5,000.00	$14,000.00	
Variable Cost	**$45.00**	$4,500.00	**$40.00**	$2,000.00	$6,500.00	
Throughput	**$45.00**	$4,500.00	**$60.00**	$3,000.00	$7,500.00	
Operating Expense					$6,000.00	
Net Profit					**$1,500.00**	
Available Min/Week					2400	
Resource A	15	1500	10	500	2000	83%
Resource B	15	1500	30	1500	**3000**	**125%**
Resource C	15	1500	5	250	1750	73%
Resource D	**25**	2500	5	250	**2750**	**115%**
Totals	70	7000	50	2500	9500	
T/Cmin (Constraint=B)	**$3.00**		**$2.00**			

With the simple modification of increasing Processing Time for Product P on Resource D from 15min to 25min, both Resources B & C are overloaded.

From the calculations above, we can see this simple change has now resulted in a situation where both Resource B and Resource D are overloaded. Since we did not change the Selling Price, Variable Cost or the Processing Time for Product P or Q on Resource B, the T/Cmin for each Product also remains the same (if we assume Resource B as the most loaded resource remain the constraint)

So will apply the Rule of "Prioritizing based on Highest T/Cmin" still work if we take the "Constraint" to be the most loaded Resource?

Modified P&Q (Considering Capacity Constraints)

Rule: Prioritize P over Q based on Highest T/Cmin using Most loaded resource as "Constraint"

	Product P		Product Q		Total	% Utilization
	Unit	Total	Unit	Total		
Demand	1	**96**	1	**0**		
Sales Price	**$90.00**	$8,640.00	**$100.00**	$0.00	$8,640.00	
Variable Cost	**$45.00**	$4,320.00	**$40.00**	$0.00	$4,320.00	
Throughput	**$45.00**	$4,320.00	**$60.00**	$0.00	$4,320.00	
Operating Expense					$6,000.00	
Net Profit					**-$1,680.00**	
Available Min/Week					2400	
Resource A	15	1440	10	0	1440	60%
Resource B	15	1440	30	0	**1440**	**60%**
Resource C	15	1440	5	0	1440	60%
Resource D	**25**	2400	5	0	**2400**	**100%**
Totals	70	6720	50	0	6720	
T/Cmin (Constraint=B)	**$3.00**		**$2.00**			

From the above calculations it is clear that if we prioritize Product P (based on Product P still having the highest T/Cmin if we considering Resource B still the constraint) this company does not make money – a loss of $1680.00/week.

The maximum number of Product P that we can make without overloading either Resource B or Resource D is 96 (there is no capacity to make any Product Q).

However, the reader will notice from the above calculations that in our attempts to prioritize P, the resource that now govern the Throughput is not Resource B but resource D, so should we change the rule and prioritize based on the highest T/Cmin considering Resource D as the Constraint?

So let’s repeat the calculations but using Resource D as the “Constraint” in calculating T/Cmin.

As can be seen from the calculations below, with Resource D considered the “Constraint” now Product Q has the highest T/Cmin of $12/min vs only $1.80/min for Product P. Now wonder prioritizing Product P delivered such a large loss…?

Can we do much better if we Prioritize Product Q now?

Modified P&Q (Considering Capacity Constraints)

Rule: Prioritize P over Q based on Highest T/Cmin using Limiting resource as "Constraint"

	Product P		Product Q		Total	% Utilization
	Unit	Total	Unit	Total		
Demand	1	**60**	1	**50**		
Sales Price	**$90.00**	$5,400.00	**$100.00**	$5,000.00	$10,400.00	
Variable Cost	**$45.00**	$2,700.00	**$40.00**	$2,000.00	$4,700.00	
Throughput	**$45.00**	$2,700.00	**$60.00**	$3,000.00	$5,700.00	
Operating Expense					$6,000.00	
Net Profit					**-$300.00**	
Available Min/Week					2400	
Resource A	15	900	10	500	1400	58%
Resource B	15	900	30	1500	**2400**	**100%**
Resource C	15	900	5	250	1150	48%
Resource D	**25**	1500	5	250	**1750**	**73%**
Totals	70	4200	50	2500	6700	
T/Cmin (Constraint=D)	**$1.80**		**$12.00**			

Again the company made a loss – this time "only" $300/week – but it's still a loss!

The interesting thing is that this time it seems Resource B is now the one that has become the limiting factor for determining how many of Product P we can produce without exceeding the available capacity of either Resource B or Resource D. The real "constraint" seem to be moving depending on the Product Mix we choose…

Three questions immediately come up:

1. With this simple change, is it possible for this company to make a profit?
2. If it is, what rule should be used in such cases (where more than one resource is overloaded based on available orders)? Or
3. Could it be that there are no simple rule for deciding on the "Optimum Product Mix" in the case where more than one "Constraint" exist and that we simply need to use a Linear Programming or even a Non-Linear Programming algorithm to solve this more complex "Product Mix" problem?

Is it possible for this company to break even or even make money?

The simplest rule we can use is that we make the same ratio of each of the confirmed order Product demand as the ratio of the "Available capacity divided by the required capacity to meet confirmed demand on the most over-loaded resource (resource B).

Calculating the Ratio to use:

From the Table with Modified P&Q (ignoring Capacity Constraint) we can see that in Order to satisfy the confirmed orders (known demand) of 100 units of Product P/week and 50 units of Product Q/week, we need:

On Resource B, 3000 min of capacity (vs available 2400 min)
On Resource D, 2750 min of capacity (vs available 2400 min)

The Ratios of Available Capacity / Required Capacity therefore:
For Resource B, 2400/3000 = 80.0% (most loaded resource)
For Resource D, 2400/2750 = 87.3%

Since we only have sufficient capacity for 80% of Product P & Q's confirmed or available demand it makes sense that as a simple rule, why not just make 80% of Product P and 80% of Product Q's confirmed demand. Note that trying to make 87% of the known demand of Product P and Product Q will exceed available capacity on Resource B.

So will this new "Product Mix" rule allow the company to not lose money?

Modified P&Q (Considering Capacity Constraints)

Rule: Make same % of each Product as Ratio of "Avail Capacity/Reqd Capacity on Most Loaded Resource")

	Product P		Product Q			%
	Unit	Total	Unit	Total	Total	Utilization
Demand	1	**80**	1	**40**		
% of Demand to Satisfy		**80%**		**80%**		
Sales Price	**$90.00**	$7,200.00	**$100.00**	$4,000.00	$11,200.00	
Variable Cost	**$45.00**	$3,600.00	**$40.00**	$1,600.00	$5,200.00	
Throughput	**$45.00**	$3,600.00	**$60.00**	$2,400.00	$6,000.00	
Operating Expense					$6,000.00	
Net Profit					**$0.00**	
Available Min/Week					2400	
Resource A	15	1200	10	400	1600	67%
Resource B	15	1200	30	1200	**2400**	**100%**
Resource C	15	1200	5	200	1400	58%
Resource D	**25**	2000	5	200	**2200**	**92%**
Totals	70	5600	50	2000	7600	
T/Cmin (Constraint=B)	**$3.00**		**$2.00**			
T/Cmin (Constraint=D)	**$1.80**		**$12.00**			

So our new Rule allows the company at least to break even, but can we find a Product Mix where the company will make money, or is the New Rule (based on ratio of available capacity vs. required capacity to meet confirmed demand) the best we can do?

Some experiments on a few simple modifications of the P&Q showed that applying the new rule to other situations where more than one resource is overloaded seems to always provide a better profit than prioritizing using "Highest T/Cmin). But more research is needed to confirm this especially considering the fact that it seems from this simply example that applying the "Highest T/Cmin" to a company with more than one overloaded resource to determine the "optimum" product mix can result in significant losses to the company when it fact a "better" product mix do exist…

So can we find a "better" product mix that will result in a profit better than the "breakeven" we achieved with the new rule?

We can answer this question (whether it is possible to make money) using a simple Linear Programming algorithm.

But before we explore this, we should recognize that if the above company was a real business where it is unlikely that customers will tolerate for long that this company cannot supply their orders, the management would need to find a way to meet the confirmed orders or risk loosing these customers.

The first strategy will be to simply "Elevate" the capacity Resource B and or Resource D with overtime or extra shifts.

If it is not possible to work overtime or add another shift, the other option is to buy additional B and D resources. However for each of these options, we will have to calculate whether the additional Throughput will exceed the additional Operating Expenses related to the overtime or additional shifts or whether the ROI for the additional machines fall within the ROI requirements of the shareholders.

In the next section we will explore the Linear Programming Solutions to both the Original and Revised P&Q to see if it provides insights both on why the normal TOC Rule does not provide the optimum for a company with more than one overloaded resources and also whether it provides insights on whether there could be a simple rule that will work for a situation where there is more than one overloaded resource (without having to use Linear Programming to solve the Product Mix every time the Demand and Resource Constraints change.

Original P&Q - Linear Programming

Problem Formulation

Step 1 - Identify the Decision Variables

Planned Production Mix = Qty of Product P (Xp) + Qty of Product Q (Yq)

Step 2 - Identify the Objective Function

Max Company Profit (Z) where **Z = (Xp * $45) + (Yq * $60) - $6000**

Step 3 - Identify the System Constraints

Demand Constraints

Demand for Product P	**$X_p \leq 100$**
Demand for Product Q	**$Y_q \leq 50$**

Capacity Constraints

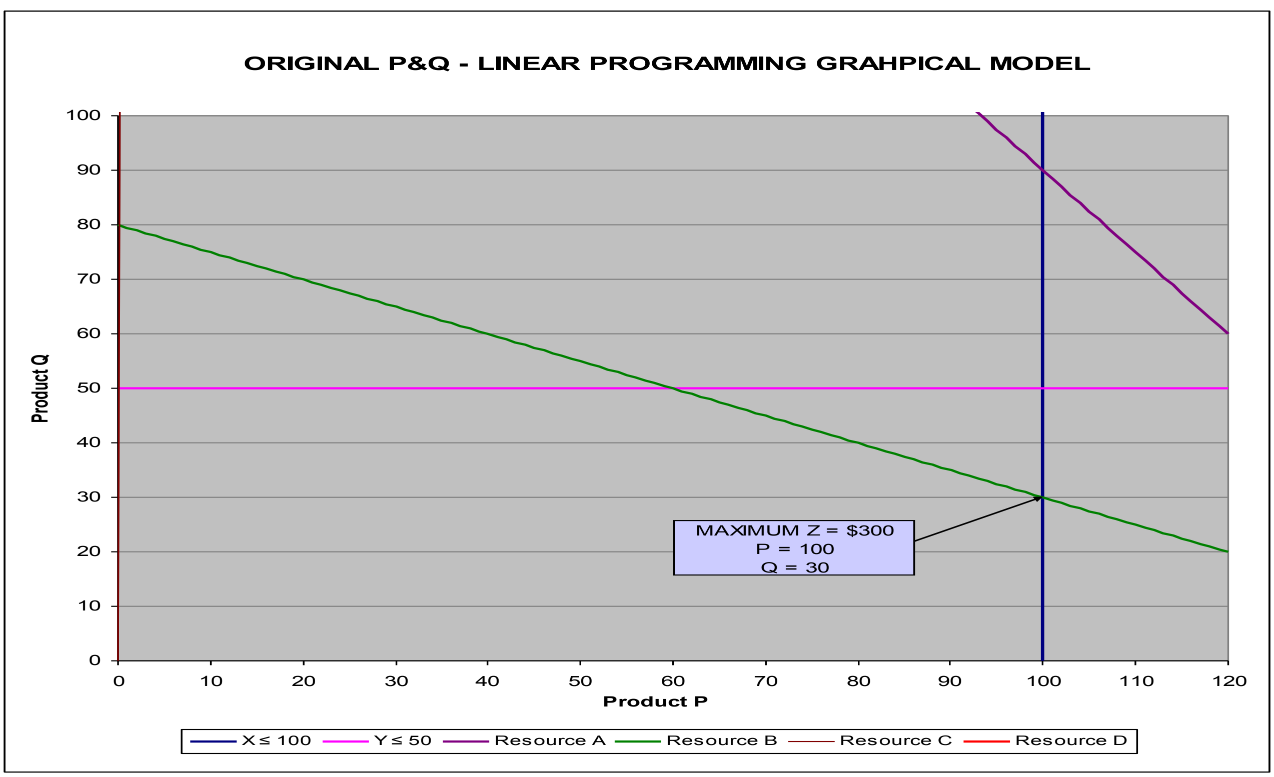
Resource A:	$15X_p + 10Y_q \leq 2400$
Resource B:	$15X_p + 30Y_q \leq 2400$
Resource C:	$15X_p + 5Y_q \leq 2400$
Resource D:	$15X_p + 5Y_q \leq 2400$

Supply Constraints

None

Solution

From the Graph below, it is clear that the Region that satisfies all the Demand and Capacity "Constraints" are bound by the two demand constraints and the Capacity Constraint of Resource B. The Maximum Profit will occur at one of the four most corner points around the boundary. In this case the maximum profit (due to the higher T/Cmin of Product P) is on the corner where the Maximum Demand Constraint for P ≤ 100 and the Resource Constraint Line for Resource B. Using simple Algebra to solve the point at which the two lines cross we find that Xp=100 and Xq=30 giving a Profit (Z) of $300/week.



Revised P&Q - Linear Programming

Problem Formulation

Step 1 - Identify the Decision Variables

Planned Production Mix = Qty of Product P (Xp) + Qty of Product Q (Yq)

Step 2 - Identify the Objective Function

Max Company Profit (Z) where **Z = (Xp * $45) + (Yq * $60) - $6000**

Step 3 - Identify the System Constraints

Demand Constraints

Demand for Product P	**$X_p \leq 100$**
Demand for Product Q	**$Y_q \leq 50$**

Capacity Constraints

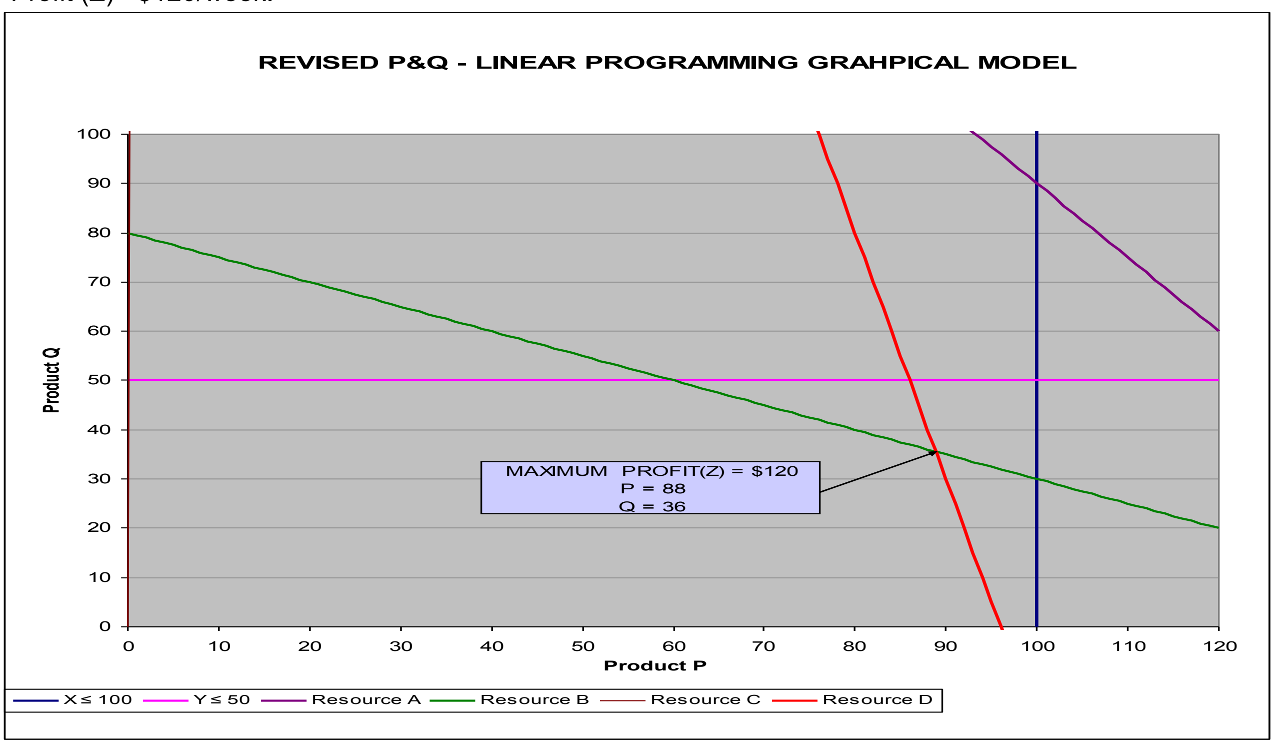
Resource A:	$15X_p + 10Y_q \leq 2400$
Resource B:	$15X_p + 30Y_q \leq 2400$
Resource C:	$15X_p + 5Y_q \leq 2400$
Resource D:	$25X_p + 5Y_q \leq 2400$

Supply Constraints

None

Solution

From the Graph below, it is clear the the Region that satisfies all the Demand and Capacity "Constraints" are bound by the demand constraint for Q ≤ 50 and the Capacity Constraints of Resource B and Resource D. The Maximum Profit will occur at one of the two corner points around the boundary of this region. In this case the maximum profit is on the corner where the lines of Resource B & Resource D crosses. Using simple Algebra to solve the point at which the two lines cross we find that Xp=88, Yq=36 which gives a Profit (Z) =$120/week.



But WHY will the point where the two Resource Capacity Constraint Lines cross give a higher profit than where the Demand Constraint Line of Product Q crosses with the Resource Constraint line of Resource D?

Conclusion

The modified P&Q example clearly shows that if the confirmed demand exceed supply capacity of more than one resource, applying the highest T/Cu to prioritize which Product Mix to make can result in less than "optimal" solution, to the extent that it can cause the company to make a loss even when a Product Mix exist that can result in a Profit.

For the more complex case of more than one resource constraint that must be considered, it seems that a Linear Programming algorithm is the only "safe" way to determine the most optimum product mix. However, considering that in practice, there can be significant variation and uncertainty in both the values of the demand and supply constraint's, a sensitivity analysis would have to be done in order to really understand how the most "Profitable" Product Mix would change based on changes in the demand and or supply constraints.

Like with most of the "Real Life" problems we experience where there are inherent uncertainty, the solution can never be to simply use a algorithm like Linear or even Non-Linear Programming. We must find a simple set of rules that acknowledge that there are normally quite a large region of "good enough" while ensuring (through a fast feedback loop) to ensure we stay well clear of the "Chaos Condition" points where not only goal units decay fast but variation and unpredictability of performance increases exponentially. It seems therefore our strategy should include first identifying the "Solution Region" and then picking a safe starting point (e.g. Product Mix) and then using a Feedback mechanism to identify when & how we should reduce and or increase the specific decision variables to get closer to an optimum value.

Lastly, it is also clear that more research needs to be done to find the simple rule(s) that can be used when a company approaches chaos conditions due to interactive constraints…

Summary of Key Insights so Far:

- After reviewing the above example with Dr. Eli Goldratt he said "(*It is clear from this simple model that*) *Using 'Highest Contribution per Scarce Resource' when more than one resource is overloaded is simply Wrong…To the extent that it can cause bankruptcy…*"
- (The example also shows that) Complexity of a system is dependent not on the number of Constraints, but the number of "interactive" constraints.
- When you do have interactive constraints, the safest "rule" seems to be to use LP to decide on the "most profitable" product mix.
- However, Considering the inherent variation and uncertainty that exist within Demand, Capacity and Supply Constraints,
    - There is a need for Protective Capacity (irrespective of which Product Mix is selected)
    - **Using LP cannot be the answer** since LP ignores the impact of the dynamic inherent variation & interdependency could also result in a "unprofitable mix" or at least in "chaotic" or unpredictable profits

- Goldratt have always indicated that when a system has **interactive constraints**, it enters **"chaos conditions"** and the "best" strategy / "rule" is simply to get out of these chaos conditions asap by "**elevating" all accept one Strategic Constraint Resource**.
- We know that the higher the variation & uncertainty, the greater the protective capacity needs to be to prevent "Chaos Conditions" (where Profits become unpredictable)
- **Question**: If it is really true that "*All complex systems are governed by Inherent Simplicity*" then there should be a (relatively) "simple" rule that we can use in the transition of getting out of chaos?
  - It seems one of the "Safe" simple rules can be to simply make an equal % of each product based on the ratio of available/required capacity (recognizing the need for sufficient protective capacity at capacity constrained resources. But will this rule always work?
  - Such a simple rule should ensure the highest total utilization of the Interactive Scarce Resources.

Appendix - Background to this Problem

I discovered the "inconsistency" that the traditional law of economics on which the TOC rule of prioritizing based on Highest T/Constraint min is based, will not give the maximum profit during July 2003 while working on a real product mix optimization problem for a multi-billion dollar High Tech based in Silicon Valley. The Company had quite a "balanced" value chain (similar capacities) which resulted in more than one resource being overloaded (capacity constraint) when the demand increased above the supply capacity.

Initially, we ignored this factor as we simply assumed that the "most loaded resource" should be used in the product mix optimization calculation for "highest T/min". However, the financial model I created showed that the change in the product mix to achieve a higher T/min would actually cause overall profitability to go down! The only hypothesis at the time was that the model's logic had an error somewhere. Once the model's logic was confirmed, I decided to use the simpler P&Q model to see if we can replicate the unexpected drop in profitability when we introduce a system with more than one overloaded resource. It was then that I discovered that an environment with "interactive constraints" fall outside the boundary conditions for the "Highest T/Cmin" law to work.

Once I realized the problem, I then developed a model to identify possible new (still relatively) simple rule of deciding the most profitable product mix based on the mix that gave the highest over-all utilization of all interactive constraints. I knew it was unlikely that this would be the optimum solution but likely that it would be a "good enough" solution for practical problems. Our strategy was to first check the theoretical profits that could be made if we prioritized based on T/Cmin using each of the overloaded resources as the Constraint. We then compared this to applying the simple load ratio rule and selected whichever of these various product mixes gave the maximum profits.

Since the company at that stage did not have access to a Linear Programming package that could cope with the complexity of their number of products and resources, since the inherent variation and uncertainty in both supply and demand was significant and lastly since forecasts showed that the increase in demand (that caused the overloaded resources) would last for only 2 to 3 months, we did not explore the LP option to find an even better product mix.

Is knowledge of this problem new?
Only after developing this new rule did I do research to find whether I was first to identify this problem. None of the other "TOC Experts" I consulted with, including Dr. Eli Goldratt, could predict that the TOC Rule in the modified P&Q I constructed would result in significant losses and secondly that in fact there was a "product mix" that would at least result in a break-even.

I discovered that the problem with the "scarce resource maximization law of economics" on which the TOC Rule was based have been identified in 2000 by the authors of "**Factory Physics**", Wallace Hopp (Nortestern University) and Mark Spearman (Georgia Institute of Technology). They used a different modification to the original P&Q example that also overloaded Resource D and B to show that using the TOC Product Mix rule based on Highest T/Cmin will result in a loss. However, their conclusion was that (using the TOC T/Cmin rule) "*can result in extremely bad solutions and that the only method guaranteed to solve these (Product Mix) problems optimally is an exact algorithm such as those used on linear programming packages.*" They further said that *"given the speed, power and user-friendliness of modern LP packages, one should have a VERY* *good reason to forsake LP for an approximate method* (such as the simple TOC rule). The gap in their analysis is that they did not:

- recognize that for specific boundary conditions the TOC rule will produce the optimum results (environments with only one overloaded resource),
- identify clearly the boundary condition where the rule will produce sub-optimum (and sometimes devastating) results (environments with interactive constraints)
- did not try to develop a simple "good enough"rule that can be used in real life siuations where variability and uncertainty in key parameters make the application of LP engines inpractical.

References:

1. Goldratt, Eliyahu M, The Haystack Syndrome, North River Press
2. Hopp & Spearman, Factory Physics 2nd Edition, McGraw Hill

The author wants to extend his deep appreciation to Dr. Eli Goldratt for his contribution to this paper.